Abstraction and Abstract Concepts: On Husserl's *Philosophy of Arithmetic*

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1. Basic questions in the contemporary theory of abstraction

When talking about abstraction theory, we might distinguish at least three questions. There is first a straightforward *metaphysical* question, namely the question as to whether there are abstract entities. *Platonism* is a position which simply maintains that there are abstract objects and *nominalism* a position which denies the existence of such entities. There is secondly a *semantical* question. One wonders whether there are utterances whose truth commits one to the existence of abstract objects. Let us call *semantical realism* a position which maintains that the truth of utterances of a certain domain of discourse[1], e.g. mathematics, commits us to the existence of abstract entities and *semantical antirealism* a position which denies that the truth of the same utterances involves any such commitment. There is finally an *epistemological* issue, namely the question as to whether there is knowledge about abstract objects. Let us call *epistemological realism (with respect to abstract objects)* a position, which maintains that we have *direct knowledge* of abstract objects (just as one might believe that we have direct knowledge of concrete objects) and *epistemological antirealism (with respect to abstract objects)* a position which maintains that we have no such direct knowledge (but, say, only inferential knowledge) of abstract entities.

Considerations of theoretical coherence might lead us to expect that

[1] I shall use expressions such as 'abstract terms' or 'abstract discourse' in a loose way in order to specify simple or complex linguistic expressions which are *generally taken* to refer to abstract entities. The same applies, *mutatis mutandis*, to my usage of 'abstract concept'. An abstract concept is a concept which is *generally taken* to be of an abstract entity. The lose usage of these terms is on purpose since the present paper is supposed to be neutral with respect to the precise ontological status of abstract objects. Notice, finally, that the fact that a concept is abstract in the present sense, i.e. a concept being about an abstract entity, does not imply as such that the concept itself ought to have a specific ontological status, whether abstract or concrete.

common philosophical positions with respect to a specific abstract discourse would combine either Platonism and semantical as well as epistemological realism or nominalism and both semantical and epistemological antirealism. These extreme positions, however, have consequences which appear far too radical. If epistemological realism with respect to the external world relies on the idea that we have perceptual access to concrete entities, it has proven much more difficult to show what sort of cognitive capacity could be involved in our direct grasp of abstract entities. Similarly, but in an opposite vein, it is hard to accept the idea that rejecting epistemological realism with respect to a certain domain of abstract objects should involve that our utterances concerning that very domain should not be taken to be true at face value or that what they purport to be talking about in fact does not exist. Important philosophical work has thus been dedicated to the elaboration of mixed positions, positions which combine either Platonism with some sort of antirealism or nominalism with some sort of realism. One variety of fictionalism, for instance, results from a combination of metaphysical antirealism and semantical realism: certain utterances do commit us to the existence of abstract entities, but they occur in a context where notions such as existence are crucially modified.

Since Frege a lot of energy has been invested into the discussion of the relation between semantical and metaphysical questions connected to abstract, mainly mathematical, discourse. Frege's approach, as it was first formulated in his *Grundlagen* (Frege 1884) and then developed in his *Grundgesetze* (Frege 1893/1903), was characterised by logicism, the idea of a foundation of arithmetic on logic. The original program, as is well known, could not be carried out. More recently, however, Crispin Wright has argued that one can find in Frege's *Grundlagen* a principle, Hume's Principle[2], which allows a partial vindication of Frege's original logicism. Indeed, it has been shown that Peano Axioms can be obtained from Hume's Principle in conjunction with suitable definitions.[3] If Hume's Principle, although no part of logic in the strict sense, has the status of an a priori and even necessary truth, then, Wright submits, "there will be an

[2] See §2 below. The terminology comes from Boolos 1987. Frege himself refers to Hume as a precursor of the principle in §63 of *Grundlagen*.

[3] Cf. Wright 1983: 154–69 and Parsons 1965. See also Boolos 1997, 245–6 and Heck 2000, 187.

priori route from a mastery of second-order logic to a full understanding and grasp of the truth of the fundamental laws of arithmetic" (Wright 1997, 210).

The interesting point about Hume's Principle in relation to our topic is that it is precisely considered as constituting the basis for the sort of abstraction by which numbers are introduced as objects into mathematical discourse.[4] It is further argued that this Fregean conception, if it can be sustained, would provide substantial means to deal with some of the most pressing problems which are believed to have saddled many traditional conceptions of our knowledge of abstract objects.[5]

In what follows I shall first rehearse some well-known results in this debate, since it builds the theoretical background against which abstraction theory has come to be discussed. My principal target in the present paper, however, does not lie so much in a discussion of the semantics of abstract discourse as in a clarification of its underlying epistemology. I wish to understand what sort of cognitive performances lie at the basis of knowledge expressed in abstract discourse and what the precise epistemic import of those performances might be. In order to do so, and for a start, I shall concentrate on Husserl's early theory of abstraction or, more precisely, on the debate between Frege and Husserl on the relevance of the study of certain cognitive performances for the determination of the nature of concepts of abstract entities, such as numerical concepts. This, I hope, will enhance the view that there exists a viable Husserlian alternative to the dominating Fregean theory of abstraction.

2. Fregean Theories of Abstraction

Two well-known principles lie at the basis of Frege's theory of abstraction: the Context Principle and Hume's Principle. As it is presented in Frege's *Grundlagen*, the Context Principle states that one should "never ask for the meaning of a word in isolation, but only in the context of a proposition"

[4] This has most prominently been argued for in Wright 1983.

[5] Cf. Wright 1997: 209. The recent debate about the epistemological challenge posed by knowledge of abstract objects goes back to Benacerraf 1973.

(Frege 1884, XXII).[6] Hume's Principle, instead, is presented in §63 of *Grundlagen* as the famous *contextual definition* of the cardinality operator, such that, for any two concepts G and F, the number of Gs is identical to the number of Fs if, and only if, G and F stand in a one-to-one correspondence relation.

A number of general, more theoretical than exegetical considerations, should be in order. First, independently of whether one takes the Context Principle as applying to any linguistic expression or to mathematical expressions only, it admits of at least two related readings, a semantical one and an epistemological one. On the semantical side the Context Principle appears to establish that the reference of a word is not determined independently of the word's contribution to the truth of the utterances in which it occurs. As such, the principle simply states a *dependence* of reference on truth. Such a dependence, however, can have an epistemological consequence when it comes to the specification of what it means to have knowledge about the entities the words under consideration refer to. In this perspective, the context principle might be considered as stating that one cannot have any epistemic access to the referent of a word independently of one's grasp of the conditions under which the utterance in which the word occurs is true. This amounts to an epistemological *priority* claim, a claim to the effect that one simply is not given an object of a certain type before one is making a judgement concerning the facts or situations of which the object might be taken to be a constituent.

As applied to abstract expressions such as numerals, the context principle would yield, on its semantical side, the thesis that what a numeral refers to (e.g. a certain number) is not determined independently of the conditions under which one or more statements in which it occurs are true. A sign, such as the numeral '11', refers to the number eleven not by itself, or by virtue of some causal relation between itself (or its instances) and the number eleven, but in virtue of a series of utterances such as '11 is greater than 10', '11 is prime', etc. Among the entities to which the numeral '11' could refer, we ought to choose the one whose nature is compatible with the truth of the utterances in which its name occurs. The natural

[6] The principle is explicitly applied in different contexts, e.g. *Grundlagen* §60, §62 and §106.

epistemological *pendant* of this view would be that there is no route to a number independent of the recognition of the truth of a certain set of utterances containing numerals. There is no independent epistemic access to the number eleven, say, against which we could assess our judgement that eleven is greater than ten or that eleven is prime. The best we can do is to move from one judgement to another and to hope that the utterances we take to be true do not commit us to refer to an entity whose nature is at odds with our deepest metaphysical convictions.

It is precisely in the light of these epistemological considerations that the extension of the application range of the context principle to *all* domains of discourse might appear off the wall. The problem lies not only and primarily in the fact that we consider ourselves to be able to check the truth of a particular judgement concerning, say, a middle sized item in our immediate environment, e.g., by looking at it, as in the fact that our metaphysical convictions concerning the external world are constrained by our perceptual involvement with it. As long as it is not shown that there is a similar constraint in the case of our beliefs concerning abstract entities such as numbers, we tend to be dissatisfied with an indiscriminate application of the Context Principle, which flattens the picture instead of articulating the expected asymmetry. As we shall see, Husserl's conception is partly characterised by the attempt to determine the sort of constraint, albeit not merely perceptual, which is at work in our beliefs about abstract entities.

Some of the utterances whose truth-conditions determine the reference of the expressions occurring in them appear to be more crucial than others. Identity statements – or recognition statements, as Frege used to call them – have a special status in this respect. Clearly, if we do not know who the expression 'John' refers to, being told that John is identical to Mary's father might be of substantial help. The statements Frege had in mind when he was discussing the contextual definition of numbers were precisely identity statements. There is, as William Demopoulos (1998, 487) has recently emphasised, a rather natural connection between Frege's context principle, which establishes the dependence of reference on truth, and Hume's principle, which provides a general schema of necessary and sufficient conditions for identity statements about abstract entities.

Let us, however, have a closer look at Hume's Principle before we return to its relation to the context principle. The first point to notice about Frege's contextual definition is that the identity statements at issue involve definite descriptions, which are obtained by application of the cardinality operator on a predicate, rather than numerals themselves. The identity statements have the form 'the number of Gs = the number of Fs' and not simply 'n = m'. This isn't really a problem, given that for any numeral η there is a predicate φ such that the statement 'η = the number of φs' is true.[7] The burden of occurring in identity statements for which we can provide sufficient and necessary conditions would thus be deferred from the cardinal to its corresponding operator.

But what about Hume's Principle itself? At first glance, Hume's principle specifies an identity criterion for numbers. Similarly, an identity criterion for fathers could be given as follows: 'the father of A is identical to the father of B iff A and B are siblings'. Or, in another famous example of Frege's: 'The direction of line *a* is identical to the direction of line *b* iff *a* and *b* are parallel'. Such identity criteria are obviously informative only under the condition that the relation stated on the right hand side (being a sibling, being parallel) is at least *conceptually* independent from the property (being the father of, being the direction of) attributed on the left hand side. Questions concerning the precise nature of the conceptual independence at issue have been widely discussed. Frege himself appears to have adopted an epistemological criterion, according to which concept C is independent from concept D if one can grasp C without grasping D (if one can have a thought involving C without having a thought involving D). One such case would precisely be given if concept C has an intuitive basis which D lacks. Thus, the concept <parallelism> is independent from the concept <direction of a line> if, as Frege submits, one can have "the representation [*Vorstellung*] of parallel lines" but not a representation of the direction of a line (*Grundlagen*, §64). The concept <direction of a line>, Frege says, "is attained through an intellectual inquiry which comes after the intuition [*Anschauung*] of the line" (ibid.).[8]

[7] For our purpose it would actually suffice to state that there is at least a definable set of numbers for which this condition obtains.

[8] If applied to concepts such as <father of> and <sibling> this criterion will hardly yield the desired conceptual independence, thus making the intended usage of the related identity criterion spurious.

We shall have to come back to the notion of conceptual independence involved in Frege's usage of Hume's Principle. But let us for the moment reflect on what we obtain from the combination of Hume's Principle with the context principle. Demopoulos thinks that "Frege's central application of the context principle … consists in the claim that knowledge of the truth of Hume's principle suffices for knowledge of reference" (488). And this, he submits, would free Frege from having to resort to the naïve Platonist picture, "according to which numbers and other abstract objects are 'ostended in intuition'" (ibid.). If I understand this claim correctly, the idea is that if one can provide *informative* identity criteria as mentioned above, where the concepts on the right hand side are independent from those on the left hand side, then application of the context principle would entitle us to the attribution of a genuinely referential character to the singular terms occurring on the left hand side even if the objects thus referred to are not available in ostension.

There are two remarks one can make at this stage. First, the strategy under consideration frees oneself from naïve Platonism only under the condition that concepts on the right hand side of the intended identity statements are not abstract concepts themselves. But now, what exactly is an abstract concept? As mentioned above, at one point Frege appears to have considered that concepts are not abstract if they are obtained from intuition as opposed to concepts which are obtained "through intellectual inquiry". But this simply removes the problem by one step. For, the Platonist too makes appeal to intuition. So, what one needs at this stage is a theory of intuition such as to determine *which* intuitions, if any, we want our concepts to be based on. Notice that this requirement has to be *added* to the requirement that the concepts on the right hand side involved in the identity statements be independent from those on the left hand side.

Secondly, being entitled to consider an abstract singular term as referential does not amount as such to the claim that we have knowledge of its referent. So, to take the example I have given above, to know that the father of A is identical to the father of B on the basis of the fact that A and B are siblings does not yet constitute knowledge of A's and B's father. It is open to dispute as to what more one needs to know in order to know A's and B's father – does one need to meet him physically? But surely one would have to provide further arguments for a conception according to which, in the case of abstract terms, entitlement to their referential use constitutes as such knowledge of their referent.

I mention these two critical points because they are at the heart, or so I shall argue, of Husserl's criticism of Frege's *Grundlagen*. Let us then look more precisely at this criticism before we return to an evaluation of the Fregean abstraction theory against its Husserlian alternative.

3. Husserls criticism of Frege in PhA

Chapters VI and VII of Husserl's *Philosophy of Arithmetic* (*PhA*) discuss approaches which intend to *define* first equinumerosity in terms of a one-to-one mapping between the extension of concepts and then the concept of number itself in terms of such a mapping. Frege's theory is brought into this more general context before being discussed and harshly criticised.

Husserl's criticism is based on general considerations concerning the status and import of definitions. Husserl maintains that "one can define only that which is logically compound" (*PhA*, 119) and that there is no possible definition of "the last, elementary concepts" (ibid.). An example of this attitude is provided in Husserl's discussion of Leibniz' alleged definition of identity in terms of substitution *salva veritate* (cf. *PhA*, 96–8). Husserl objects to such a definition that it reverses the order of explanation. Two "contents" A and B, says Husserl, are not identical because they are substitutable *salva veritate*, but they are substitutable *salva veritate* because they are identical.[9]

Husserl thinks of definitions, in the present context, as providing what one might call an *epistemological* analysis of concepts. Husserl appears to require that definitions ought to satisfy at least two conditions, the *explanatory condition* and the *knowledge condition*. The explanatory condition requires, in line with what we saw above, that the *definiendum* should be explainable in terms of the *definiens* and not the converse. The knowledge condition establishes that possession of the concepts consti-

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[9] This formulation, of course, leaves room for misunderstanding, in part because it appears to mix up terms with their referents. One, but only one way of making it more precise could be: the referents of the terms A and B are identical if A and B are always substitutable *salva veritate*.

tuting the *definiens* should provide sufficient conceptual equipment for attaining any body of knowledge (with the exception of the definition itself) in which the concept constituting the *definiendum* is involved.[10]

This second condition highlights the fact that Husserl's considerations on the nature of definitions of concepts has to be viewed in an epistemic rather than in a merely psychological context. To see the point consider the difference between the conditions under which a subject could be knowing *p* while not knowing *q* and the conditions under which a subject could be accepting *p* while not accepting *q*, where *q* is defined in terms of *p*. For a definition of *q* in terms of *p* to satisfy the knowledge condition, it must be the case that a subject who knows that *p* thereby knows that *q* even if the subject might remain psychologically neutral with respect to *q*. This would typically be the case when the subject does not know that *q* can be defined in terms of *p*.

It is when it comes to primitive concepts, of which no definition can be given, that Husserl resorts to the method he used to call, like many of his contemporaries, *psychological analysis*.[11] Psychological analysis has several features, but two of them are particularly relevant in our context. In relation to primitive concepts such as <identity> and <similarity>, <whole> and <part>, <unity> and <multiplicity>, Husserl writes:

> What one can do at most in such cases is to determine the concrete phenomena from which or on the basis of which they [*the simple concepts*] are abstracted and to clarify the nature of this abstraction process; one can, where it is necessary, sharpen the boundaries of the concepts under consideration through different descriptions in order to prevent confusions with related concepts. What one might reasonably expect from the linguistic description of such a concept (for instance in the presentation of a science which is based on it) ought thus to be fixed as follows: the description must be such as to put us into the correct disposition to determine the intended abstract moments in inner or outer intuition [*Anschauung*]

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[10] In a slightly more precise way: Is *c* the *definiendum* and *p* the body of knowledge in the acquisition of which *c* is involved, then the concepts $k_1,\ldots k_n$ provide a definition of *c* only if possessing $k_1,\ldots k_n$ provides sufficient conceptual equipment for gaining knowledge that *p*.

[11] Remember that Husserl's *Habilitationsschrift*, which lies at the basis of PhA, was originally called: *Über den Begriff der Zahl. Psychologische Analysen*. Among the authors using the same expression at Husserl times are: Alexius Meinong, Carl Stumpf and of course Franz Brentano.

> or to reproduce in ourselves the mental processes which are required for the formation of the [*abstract*] concept (*PhA*, 119).

The psychological analysis of abstract primitive concepts ought to satisfy both a descriptive and what might be called a psychological requirement. It should first describe the concrete phenomena (the mental states or events having a concrete content) from which the concepts are abstracted, and the abstraction process thus involved. In order to be psychologically appropriate, such a description should further put the subject understanding it in a position to mimic or reproduce in his own mind the process leading to the abstract concepts. Although in obvious need of further clarification, this last requirement has the effect of eliminating a whole cluster of descriptions which might otherwise appear acceptable. So, for instance, a purely neuro-physiological description of the abstraction process, as it is implemented in the brain, even if available and true, would not satisfy the psychological requirement. For one will hardly grasp an abstract concept by understanding how the brain, or for that matter any other physical system, implements the abstraction process leading to it.

Psychological analysis, and especially Husserl's conception of it in the *PhA*, has been the object of severe criticism. It is often taken to be at the very heart of the sort of psychologism for which Frege showed much contempt and from which Husserl is said to have finally distanced himself. This is not the place to revisit this story, which we know to be much more complex than what we were made to believe for many years. I want to suggest, however, that we separate in the discussion to follow the general methodological relevance of psychological analysis with respect to abstraction theory from the acceptance of Husserl's concrete proposals concerning the process of abstraction and the concrete phenomena on which it is said to be based. Indeed, objections to the latter have often constituted an illicit ground for the rejection of the former.

In light of these considerations Husserl's criticism of Frege's concept-tion of numbers should now become understandable. Although Husserl himself is not always careful in distinguishing them, his remarks ought to be assembled in at least two different groups. On the one side there are those remarks which are aimed against the attempt to provide a definition of number. On other side there are remarks which are intended to show that

Frege's approach would not be of much help even if taken, certainly against Frege's own intention, from the point of view of a psychological analysis of the primitive concept <number>.

Let me start with Husserl's comments concerning the definition of number. Husserl readily recognises that any one-to-one mapping provides a necessary and sufficient condition for the equinumerosity of two sets (cf. *PhA*, 105, 110). He rejects, instead, the idea that one could define the concept of number in terms of a one-to-one mapping. As far as I can see, he has three main arguments in favour of his claim.

First, he thinks that one can know that the elements of two sets stand in a one-to-one mapping without knowing that the two multiplicities ("*Vielheiten*") are equinumerous. Husserl notes that we might on occasion *verify* the fact that two multiplicities are equinumerous by putting their elements in a one-to-one relation with each other, but that this operation, he says, "is neither always necessary nor does the essence of the comparison [*leading to the recognition of the equinumerosity*] lie in it" (*PhA*, 99). In this first argument, then, Husserl is simply making the point that even in the case of arithmetic, what one knows is not determined by how one might verify what one knows. This being so, the concepts involved in one's specific knowledge cannot be *defined* in terms of the concepts involved in the application of a given verification process.

It is in this context, I should think, that one ought to place Husserl's remark that by establishing the presence of a one-to-one mapping between two sets one might detect a necessary and sufficient condition for equinumerosity but not the identity of the cardinals themselves (*PhA*, 104–5). As we saw in relation to Frege's approach, knowing that two sets stand in a one to-one mapping simply does not suffice for knowledge of the attributed cardinality.

Second, after having established that for the elements of two sets to stand in a one-to-one relation to each other just means for the sets to be equivalent, Husserl maintains that the two relational concepts <has the same number as> and <is equivalent to> have the same extension, but not the same "content" ("*Inhalt*", *PhA* 115–6). The argument he gives in favour of this claim is that if the two concepts had the same content, then "the attribution of a number would not be about the presently given set, but about its relation to some other set" (*PhA*, 116). This, Husserl submits, would be absurd, because when we attribute the number four, say, to the nuts on the table, we are not interested in the fact that the set under consideration belongs to a set of infinite equivalent sets, but in the fact that "there is one nut, and one nut, and one nut, and one nut" (ibid.).

One might think that the latter judgement – that there is one nut, and one nut, and one nut, and one nut – simply amounts to putting the set of nuts under consideration in a one-to-one relation to a specific set, namely the *representative* set of four '1's (four instances of the numeral '1').[12] But that would not do. There is no reason to consider the set of four '1's as being more suited for constituting the content of the attribution of the number four than any other set of four elements. If the set of four oranges does not suffice for the purpose at stake, then the set of four '1's will not do either (cf. *PhA*, 117).

The tendency to believe that the set of '1's would somehow play a distinguished role with respect to the attribution of numbers probably originates from the fact that numerals seem to be somehow more general than nuts and oranges. But, as Husserl points out, this is simply an illusion. Husserl famously suggests that what we in fact want to say, when we say that the numeral '1' applies to any single item, is that the item falls under our most general concept ("*allumfassender Begriff*", *PhA*, 117), namely the concept <something> ("*Etwas*"). On pain of an endless regress, one cannot say that for a nut, an orange or for an instance of the numeral '1' to be something means for those items (or for the sets containing those singular items) to stand in a relation to a nut, an orange or the numeral '1' (or to the singletons containing just one nut, or one orange or one instance of the numeral '1')! This appears to be the main reason why Husserl claims that <something>, just as much as <number>, is a primitive concept.

I will not be able to reconsider at present Husserl's conception of <something>.[13] We arrived at it in the context of Husserl's second objection because of the argument allowing him to dispense with the idea of a distinguished set of '1's as being the set on which our attribution of numbers would finally have to rely. Remember, Husserl's point was that

[12] It would of course be wrong to attribute this objection to Frege. Husserl himself mentions it while criticising Stolz and not Frege (cf. PhA: 114ff.).

[13] For an excellent summary of Husserl's view see Willard *forthcoming.* See also Soldati 1994: Chapter 2.

when we judge that there are four nuts on the table we are not judging that the set – or rather: the multiplicity, as Husserl says – of nuts stands in a one-to-one relation to some other set or multiplicity. We simply judge that there is one nut, and one nut, and one nut, and one nut.

Some confusion might originate from the fact that, as pointed out by Michael Dummett (1991, 142), Husserl – just as much as Frege – is not careful in distinguishing the definition of the cardinality operator in terms of equinumerosity from the definition of equinumerosity in terms of one-one mapping. But it is fairly obvious that one should interpret the argument under consideration as containing three different steps in Husserl's mind (cf. *PhA*, 104). We first establish the cardinality of a multiplicity: we then compare the cardinality of two different multiplicities in order to establish whether they are equinumerous. And we finally might *verify* – as Husserl would say – the equinumerosity by examining whether the multiplicities stand in a one-one mapping. The previous argument was aimed against the idea of *defining* equinumerosity in terms of a one-one mapping. The present argument, instead, challenges the idea of reducing the attribution of cardinality to equinumerosity *understood as* one-one mapping. So the point in this second argument is that *if* we consider equinumerosity as being defined in terms of one-one mapping, then the attribution of a certain cardinality turns out to have the wrong, namely a falsely relational content. This of course does not imply that Husserl rejects the much more fundamental requirement that for a subject to attribute a given number to a set involves the capacity on the subject's side to decide whether the set under consideration is equinumerous to some other set.

Husserl's third criticism of Frege's definition of number in terms of one-one mapping is that it generates a definition of the extension, but not of the intension (cf. *PhA*, 122) ("the content", as Husserl says) of the concept. In a footnote Husserl argues that even if, following Frege's account, the concepts expressed by locutions such as 'the number of Jupiter's moons', 'equinumerous to the concept <Moon of Jupiter>' and 'set from the equivalence class determined by the extension of <moon of Jupiter>' all have the same extension, they cannot possibly have the same content. Stated as such this point simply begs the question on what one ought to expect from a proper definition and Husserl does not really provide any new argument in favour of his claim. This third criticism might thus best be seen as an upshot of the two arguments given above.

As I mentioned above, a number of remarks Husserl makes in his criticism of the definition of number in terms of one-one mapping do not really concern the project of the definition itself, but the question as to whether considerations presented in favour of that sort of definition could be used for the purpose of a psychological analysis. Husserl's discussion of the three ways we might come to judge that two multiplicities are equinumerous (cf. *PhA*, 104–5) has to be placed within this context. Husserl notices that the most natural way to establish equinumerosity is not to determine a one-to-one mapping, but simply to count the members of the sets under consideration. As Dummett puts it, "Husserl's thesis closely resembles the answer that a child would give when first asked the question, 'What does it mean to say that there are just as many nuts as apples in the bowl?'; almost any child will reply, 'It means that, when you count each of them, you will get the same number'" (Dummett 1991, 144).

Dummett's comment has a vaguely disparaging tone to my ears, but Husserl might not have felt offended by it at all. In fact, Husserl precisely wants to say that we have an intuitive, fairly primitive access to the notion of equinumerosity, especially with small sets, which does not presuppose grasp of the notion of a one-one mapping. In his review of Husserl's *PhA*, Frege had objected that "the author forgets that counting itself rests on a one-one correlation, namely between the number words from 1 to *n* and the objects of the set" (Frege 1894, 319). Dummett thinks that this "retort is evidently wholly justified" (cf. Dummett 1991, 144).

Frege's objection might appear correct if one has in mind the ascription of a numeral to a given set. In order to *call* the number of nuts on the table '4', one needs to establish a correlation between a conventional sequence of cardinal numerals and the set of nuts under consideration. But unless one is prepared to fall back on the idea of a distinguished set of '1's to which each multiplicity should be made stand in a one-one mapping for a cardinal concept to be applied to it, the objection ought to be resisted.[14] To use an analogy Dummett himself introduces in the present context: even if one needs to have a standard of comparison in order to determine the

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[14] On counting see for instance Parsons 1994: 146ff; Heck 2000: 193ff. and 196–9; and more generally Gelman & Gallistel 1978.

numeral which has to be applied to the length of a material object, this does not imply that one somehow needs to establish a mapping between that object and the Paris standard in order to consider the object as having a determinate length. If you do not know what it means for something to have a certain length, then mapping it onto the Paris standard will not be of much help.

It is often argued that the fundamental insight in Frege's approach lies in the recognition of the fact that numbers are applied to concepts and not to objects. This is sometimes glossed as implying that it does not really make sense to ask how many objects there are, one should rather ask how many objects fall under a certain concept. You do not wonder how many things there are on the dance floor, you may either ask how many dancers or how many couples there are. Depending on the concept you apply, you will get different answers to the 'how many' question. Demopoulos thinks that, given this assumption, "it is then only a very short step to the conclusion that the content of a statement of numerical identity consists in the predication of a relation to concepts" (1998, 484).

Husserl himself was so much aware of the importance of this assumption in Frege's conception of numbers that he dedicated nearly the entire chapter IX of the *PhA* to its discussion. Husserl finds himself agreeing with the fact that the application of different concepts may yield different counting results. But he does not accept the conclusion that numbers should be considered as being attributed to concepts rather than to multiplicities.

As far as I understand Husserl's line of criticism regarding this point, he is concerned with the question of the relation between sortal concepts and the concept <something>. More precisely, he is concerned with the question as to whether sortal concepts play a more fundamental role with respect to the attribution of number than the concept <something>. Husserl writes:

> … when we count a set of objects of the same kind, for instance A, A, and A, we start by abstracting from their material ("*inhaltlichen*") constitution, thus also from the fact that they are objects of the kind A. We generate the multiplicity form ("*Inbegriffsform*") one, one, and one and observe subsequently that each one should have had the meaning 'an A'. (*PhA*, 166)

Husserl's wording in this passage is disingenuous. The main idea behind the passage, however, appears to be that one cannot even start to count the entities falling under a concept if one does not previously have a concept of being something which falls under a concept. It is not as if the concept <horse>, say, brings automatically with it the concept of being an object which falls under it. Rather, the concept <horse> generates a selection among the range of things susceptible to falling under a concept. Unless a concept applies to *things*, to entities we might count following the method 'something, and something and something', it will not generate any possibility of counting and so will not bear any numerical property.

4. Fregean abstraction theory and its Husserlian alternative: evaluation

Obviously I have been able to consider only a very partial selection of the several issues which ought to be kept in mind in order to obtain a full assessment of the Husserlian alternative to the Fregean abstraction theory. It might still be useful to draw some conclusions.

An epistemologically satisfying theory of our concept of number should satisfy at least two requirements. It first ought to provide an account such that the numerical concepts we are supposed to obtain by abstraction are grounded in concepts which are both more fundamental or primitive and psychologically suited for the process of abstraction to take place. Secondly, the process under consideration ought to be appropriate, in relevant circumstances, for constituting genuine knowledge of the abstract entities our concepts are supposed to be about.

On Husserl's view Frege's approach does not satisfy any of these two requirements. It does not satisfy the first requirement because it aims at providing a definition where a psychological analysis is needed. Husserl does not suggest that the psychological analysis one might propose can be used as a definition of the abstract concepts. But he does suggest that our grasp of abstract concepts, such as numerical concepts, ought to be grounded in more fundamental and intuitive concepts, such as the concept <something> we apply when faced with small multiplicities.

This alone might suffice for the charge that Frege's approach does not satisfy the second requirement. If we do not know how to reach abstract concepts, the evaluation of judgements involving them appears at least problematic. We have seen, however, that Husserl provides further arguments to show that Frege's definition cannot count as epistemologically informative.

It is true that Husserl's requirements on definitions in the present context are very high. It might be argued that no definition can do justice to those Husserlian requirements. Even if true, it should be kept in mind that it was not Husserl's idea that one should look at the definition of number in order to determine how knowledge of abstract objects can be possible.[15]

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[15] My interest in abstraction theory goes back to 1990, when I had the opportunity to talk about different issues in the philosophy of mathematics with Moritz Epple in Tübingen. I would like to thank him for the many things I learned from him at that time. The present version has profited from comments by Davor Bodrozic, Taylor Carman, Kevin Mulligan and Dallas Willard.

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